

An investigation of physics-informed neural network (PINN) method for estimation of reservoir permeability using poroelasticity principle

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1. Introduction

Geological CO₂ storage (GCS) in subsurface aquifers has been proposed as a near-term solution for reducing CO₂ emission. In GCS projects, understanding reservoir permeability structure is important for the designing of CO₂ injection profile and the modeling and prediction of the flow behavior of injection CO₂. However, obtaining of the reservoir permeability profile is not easy. From conventional well testing methods based on pressure measurement, only the discrete permeability at selected depth intervals can be estimated.

The distributed fiber optic strain sensing (DFOSS) tool now can accurately measure rock deformation (e.g., $1 \mu\epsilon$) induced by pore pressure change along a vertical wellbore with a remarkably high spatial resolution (\sim cm). Since the pressure change and the deformation are near-linearly related under small perturbations (e.g., the Biot's static poroelasticity principle), the strain change contains the information of the pressure change and reservoir permeability. Our previous study (Zhang et al. 2021) has demonstrated that the distributed strain data can be used for inversely estimating fine-scale reservoir permeability structure.

Albeit the success, in the study, there was a problem that the forward run using the finite element method is quite slow. This makes it hard to do inverse modeling and apply the method to more complex models. In this project, we aim to investigate the Physics-informed deep learning (or physics informed neural networks, PINNs) method for solving the inversion problem and estimate the reservoir permeability structure from distributed strain data. PINNs are neural networks that are trained to satisfy physical equations, such as the poroelasticity equation that relates pressure and deformation in porous media. By using PINNs, we may be able to avoid the expensive and slow forward runs of the finite element method, and instead use the neural network to simultaneously fit both the equations and strain data. The main steps of our method are:

Step 1: Construct the PINNs based model for poroelasticity equations using SCIANN libs (e.g. a TensorFlow wrapper for PINNs).

Step 2: Benchmark the PINNs poroelasticity model with the analytical solution of

Barry-Mercer fluid injection problem.

Step 3: Modify the PINNs model to cylindrical coordinates and perform the modeling for a water pumping problem with the synthetic distributed strain data. Minimize the loss function of the PINN, which consists of the mean squared error between the network output and the physical equation, the data mismatch term. Obtain the estimated permeability structure from the trained PINN.

2. Mathematical equations

The equations of Biot's quasi-static poroelasticity for the hydraulic and mechanical problem are as follows:

$$\frac{\partial}{\partial t} \left(\frac{1}{M} \frac{\partial p}{\partial t} \right) + \nabla \cdot \left(\frac{k}{\mu} \nabla p \right) = 0$$

$$\nabla \cdot \left(\frac{1}{M} \frac{\partial p}{\partial t} + \frac{b}{M} \nabla \cdot \mathbf{u} \right) = 0$$

where p is the fluid pressure, M is the Biot modulus, k is the permeability, μ is the fluid viscosity, b is the Biot coefficient, \mathbf{u} is the displacement.

Following Haghighat et al. (2022) and Amini et al. (2023), in the non-dimensional forms (in cartesian coordinates), the above equations can be written as:

$$\frac{b^2}{\overline{K}_{dr}} D^* \frac{\partial \overline{p}}{\partial \overline{t}} + b \frac{b}{\overline{K}_{dr}} D^* \frac{\partial \overline{\sigma}_v}{\partial \overline{t}} - \frac{\overline{k}}{\overline{\mu}} \nabla^2 \overline{p} - f = 0$$

$$\overline{K}_{dr} \nabla \cdot \overline{\epsilon}_v + \frac{1}{2} \frac{1-2\nu}{1+\nu} \overline{K}_{dr} \nabla \cdot (\nabla \cdot \overline{\mathbf{u}}) + \frac{3}{2} \frac{1-2\nu}{1+\nu} \overline{K}_{dr} \nabla \cdot (\nabla \overline{\mathbf{u}}) - b \nabla \overline{p} = 0$$

where the overline denotes that the variable is dimensionless, \overline{K}_{dr} is the drained bulk modulus, $\overline{\epsilon}_v$ is the dimensionless volumetric strain, f is the body force term. For the aquifer well testing setting in this study, we rewrite the hydraulic and mechanical equations to the forms in cylindrical (2DRZ) coordinates.

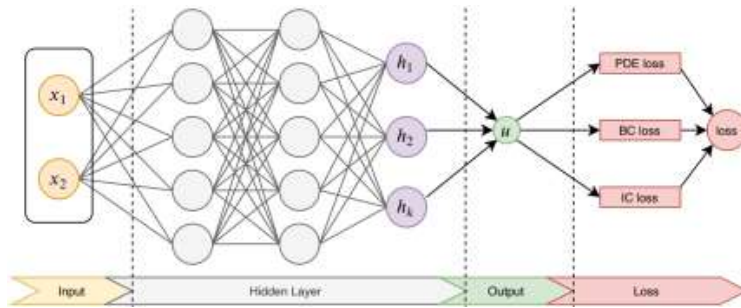


Figure 1. A schematic of a PINN for solving general PDEs (Peng et al. 2021).

The solution space of the PDEs are approximated using deep neural networks (Figure 1). Considering a PDE $\mathcal{P}u(\mathbf{x}, t) = f(\mathbf{x}, t)$ in the domain $\Omega \times T$, by PINNs, the variable $u(\mathbf{x}, t)$ is approximated by a set of deep neural networks as $u(\mathbf{x}, t) \approx \tilde{u}(\mathbf{x}, t) = \mathcal{N}_u(\mathbf{x}, t; \boldsymbol{\theta})$. We put the hydraulic and mechanical equations and initial/boundary conditions as the

constraint (or the loss function) terms of the deep neural networks. The loss function is:

$$\begin{aligned} \mathcal{L}(\mathbf{x}, t; \boldsymbol{\theta}) = & \lambda_1 |\mathcal{P}\tilde{u}(\mathbf{x}, t) - f(\mathbf{x}, t)|_{\Omega \times T} \\ & + \lambda_2 |\tilde{u}(\mathbf{x}, t) - g_D(\mathbf{x}, t)|_{\Gamma_D \times T} \\ & + \lambda_3 |\tilde{q}(\mathbf{x}, t) - g_N(\mathbf{x}, t)|_{\Gamma_N \times T} \\ & + \lambda_4 |\tilde{u}(\mathbf{x}, t_0) - h(\mathbf{x})|_{\Omega \times T_0} \end{aligned}$$

where λ_i is the weight of each term (the PDE, Dirichlet and Neumann boundary conditions, and initial condition). The training process is to find the “best-fit” approximated solution through minimizing the total loss function on the sampling points. Using the approach proposed by Haghighat et al. (2022), we sequentially train the hydraulic and mechanical networks to let the networks approximate the coupled hydro-mechanical system and improve the convergence. We have tested three problems: (1) the time-dependent, uniform, Barry-Mercer fluid injection problem; (2) a synthetic, time-independent, layered aquifer fluid pumping model; and (3) a synthetic, time-dependent, layered aquifer fluid pumping model.

3. Challenges and Results

(1) Barry-Mercer fluid injection problem

Referring to Amini et al. (2023), we set the fluid injection/production well at point $(x=0.25, y=0.25)$, and we use “observation” data at point $(x=0.75, y=0.75)$. We use a structured sampling grid and sequential strategy to train the problem. The results (Figure 2) reveal that the PINNs can well model the fluid injection/production resulted mechanical responses. Using the displacement or strain records, the permeability of the media can be inversely estimated with the training process.

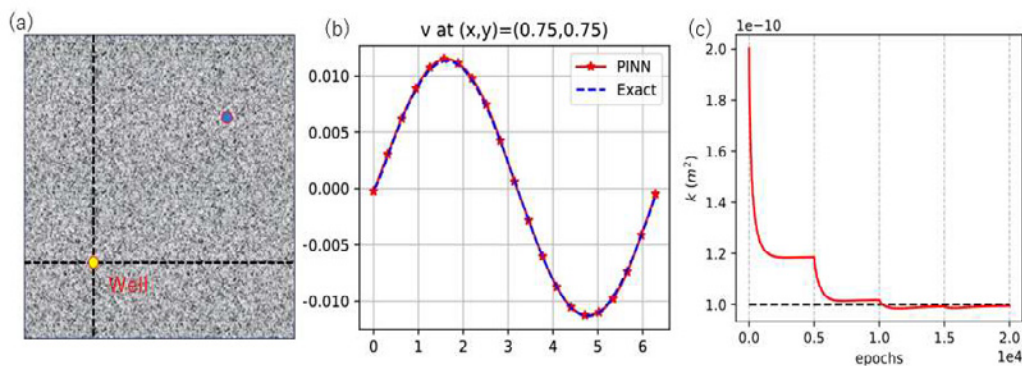


Figure 2. (a) The setting of the Barry-Mercer fluid injection problem with an injection well and observation point. (b) The PINNs modeled vertical displacement can be well matched at the observation point to the analytical solution. (c) The inversely estimated permeability as a function of training epochs.

(2) Synthetic aquifer testing model (steady state)

Next, we test the PINNs for a synthetic 2D cylindrical (2DRZ) aquifer water pumping model. The model is similar to those in common aquifer testing setting, but with an observation well installed with a distributed strain sensing tool, which can provide the entire profile of the vertical strain along the well, representing the aquifer strain. In the model, we divide the target formation (ROI) as 80 layers with different permeability values. To reduce the model training difficulty, we first only consider a steady-state case, in which no temporal change is considered. The results (Figure 3) suggest the PINNs model can well estimate the permeability profile for most parts of the target formation except those near upper and lower boundaries, where large errors exist.

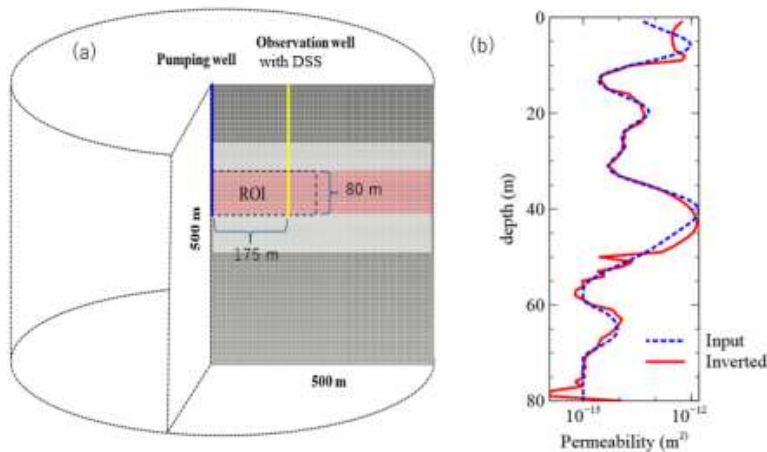


Figure 3. (a) The setting of the aquifer water pumping problem with a pumping well and an observation well (Zhang et al. 2021). (b) The inversely estimated permeability profile compared with the input permeability profile for synthetic model.

(3) Synthetic aquifer testing model (time-dependent)

The model setting is same as case (2) but consider the time-dependent changes. For this case, although we have applied the techniques such as the sequential training, nondimensionalization and adaptive weight strategy, we are not successful in the training process for time-dependent and heterogeneous field model. We find that the training process for time-dependent heterogeneous parameter field model of PINNs is a non-trivial optimization task. The loss of the training process is very unstable. The reason may be because we use less sampling points. However, increasing sampling points make the training very time consuming and become impractical for our purpose. I am still making effort to improve the training process by changing the network architecture or using transfer learning.

4. Conclusions

In this short report, we presented our preliminary results using PINNs method for modeling the hydromechanical coupled problem and parameter estimation. We find the PINNs method can accurately fit the simple uniform model. A useful characteristic of PINNs is that it likes a many-dimensional fitting problem--it not only fits the PDEs but also simultaneously fits the parameter with the measurement data. Therefore, an inverse modeling can implicitly and one-shot solved within the same training process of the PINNs. This way greatly benefits the inverse modeling compared with conventional gradient-based inverse modeling approaches, which needs many times of forward modeling and construction of Jacobian matrix, explicitly.

However, for completed heterogeneous model, this study finds that the training process is quite difficult for using the current network architecture. In addition, the training process is not transparent and readily controlled as in common gradient-based inversion methods. More investigations are needed to improve the network setting and training techniques.

Acknowledgements

This paper is based on results obtained from a project (JPNP18006) commissioned by the New Energy and Industrial Technology Development Organization (NEDO) and the Ministry of Economy, Trade and Industry (METI) of Japan. This research is partially supported by Initiative on Promotion of Supercomputing for Young or Women Researchers, Information Technology Center, The University of Tokyo.

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